Topic Study Group 20

Learning and Cognition in Mathematics (Including the Learning Sciences)

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ABSTRACT This paper includes the themes and descriptions for TSG-20 Learning and Cognition, the session topics developed in response to the themes, a report of the review process and results of this, the TSG-20 program including the research focus for each invited speaker, profiles of the invited researchers, the content of their submitted papers, and the authors and titles of other papers. It also includes participant reflections about sessions that provide indicators of future intended research directions.

Keywords: Learning; Cognition; Teacher practices; Student activity.

1. The Theme, Subthemes, and Descriptions

The scope of research on learning and cognition in mathematics education is extensive and diverse in relation to questions posed, theoretical frameworks selected, and methodologies employed. Theoretical perspectives include (but are not limited to) forms of cognitive constructivism, and social constructivism, and more recently, interconnections between these (including integration, partial integration, and networking of such theories). Affective, and embodied elements, and personal characteristics of learners and teachers are amongst the many other constructs that form part of various theoretical frameworks. Learning and Cognition in Mathematics, TSG-20, 2020 specifically included 'the Learning Sciences' which interrogates interplays between cognitive, social, psychological and cultural elements of learning processes in diverse contexts, for the purpose of 'improving' learning environments. Research into learning mathematics through STEM (Science Technology Engineering and Mathematics) Education although increasing, is not yet reflected in the proportion of STEM related papers submitted to TSG-20 in ICME14. Although this description contains illustrations of research foci within TSG-20, there are opportunities for intending contributors to focus within these, or to justify other foci associated with learning and cognition in mathematics education.

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1.1. Subtheme 1: teacher change processes and influences upon them

Processes of teacher learning, that can enhance student learning of mathematics, have been researched with various theoretical frameworks employed (including but not limited to cognitive, social, belief based, and dispositional frameworks, and interconnections between various of these frameworks). Areas of research into influences on teacher change processes include but are not limited to professional learning models employed, types of pedagogical approaches under focus: teacher-controlled or teacher-guided learning, and approaches enabling various degrees of student autonomy in the learning of mathematics. Depending on the personal characteristics of the teacher, some approaches may be easier to employ than others. Such personal characteristics include but are not limited to types of knowledge possessed, prior experiences, and whether resilience and/or self-efficacy are possessed. There are many other factors associated with teaching that could become the focus of a submitted paper, as long as that focus can be justified as belonging to TSG 20 Learning and Cognition.

1.2. Subtheme 2: student learning processes and influences upon these

Theoretical frameworks employed to study processes of student learning of mathematics include, but are not limited to, those associated with cognitive constructivism, social constructivism, and embodied, cultural, and material conceptions of mathematics cognition. Various combinations of these theoretical framings have also been developed. Influences on the nature of mathematical understandings developed include, but are not limited to, the degree of student autonomy in the learning situation, affective elements of the process, the nature of the learning environment, and personal characteristics of the student. Study of processes associated with the construction of mathematical insights are crucial to this theme because they can build deep mathematical understandings and positive student personal characteristics. Study of learning processes in situations in which students have little to no autonomy is also important as many mathematics teachers employ such pedagogical approaches. Studies of how to increase students' feelings of safety in such controlled learning situations or decrease the boredom of other students are important areas of research, as is study of learning in particular mathematical situations.

1.3. Subtheme 3: the learning sciences

The Learning Sciences is dedicated to furthering the scientific understanding of learning processes for the purpose of designing and implementing learning innovations to increase learning opportunities. This research field highlights the social nature of learning and the many different settings in which learning may occur. Studies interrogate various interplays between cognitive, social, psychological and cultural factors in learning processes in diverse contexts. To enable study of learning as it occurs in messy naturalistic settings, creative research designs have been, and continue to be, developed.

Studies in this multidisciplinary field include but are not limited to foci such as: the situated nature of knowledge and ways of knowing and learning; individual, and group learning processes; and mathematics learning in out-of-school settings, such as, museums, and homes; mathematics learning difficulties and disabilities. Student learning and teacher learning are two of the many areas of researcher attention within this field. Further study of mathematics learning in out of school settings has the potential to inform mathematics learning more generally.

2. Organizing TSG-20

2.1. Panel

The Study Topic Group, TSG-20 Learning and Cognition included the following panel members, all of whom contributed in various ways to the development of the TSG:

Chair: Gaye Williams, University of Melbourne;
Co-chair: Pablo Dartnell, University of Chile;
Members:
Wenjuan Li, New York University,
Zain Davis, University of Cape Town, and
Chunli Zhang, Beijing Normal University.

This team was drawn from universities in various countries across the world. All five team members took part in the review process. Gaye Williams, Pablo Dartnell, and Wenjuan Li hosted the three sessions of TSG-20 at ICME14.

2.2. Invited speakers

Selection of invited speakers was guided by the themes of focus. These speakers were Alison Castro Superfine (USA), Keiko Hino (Japan), Lieven Verschaffel (Belgium) and Alejandro Maiche (Uruguay). However, Alejandro was unable to participate due to circumstances at the time.

2.3. Review process

25 submissions were received for TSG-20. The 4 from invited speakers were accepted without review. One was a poster, and one paper was referred to another TSG for which it was more appropriate. The other 19 were research papers. The 5 TSG-20 panel members undertook the reviews. Each paper was reviewed by 2 panel members (one chair/cochair and another panel member). Where there was disparity between these two reviews, the other chair/co-chair also reviewed the paper before it was discussed by the chair and cochair. Where a paper was close to being judged a long paper, authors were provided with advices and invited to resubmit their papers before a final judgement was made. 14 papers had two reviewers, and 5 papers had 3 reviewers. The review process classified the 19 contributions as 5 long papers and 14 short papers, and authors from 12 of these 19 papers accepted and presented in TSG-20. The 12 presentations, together with the invited talks, were set into 3 sessions and listed in Tab. 1, in which IT stands for invited talks, LO for long oral presentations, and the others are short oral presentations.

Tab. 1. List of presentations presented in TSG-20

Paper and author(s)
Session 1
[1] Exploring new models for teacher professional learning: Working *with* teachers rather than on. ***Alison Superfine Castro*** (USA). (IT)
[2] Introduction of STEM education through collaborative action research practices. ***Fatlume Berisha*** *and Eda Vula* (Kosovo). (LO)
[3] Theorizing teachers' learning of students' mathematical thinking in the context of student-teacher interaction. ***Biyao Liang*** *and Kevin C. Moore* (USA). (LO)
[4] Students' ways of thinking in a computer-based mathematics investigation project. ***Joyce Mgombelo****, Wendy Ann Forbes, Chantal Buteau, Eric Muller* (Canada)*, and Ana I. Sacristán* (Mexico).
[5] Reciprocity between teachers' and students' problem-solving actions enables teacher change. ***Gaye Williams*** (Australia).
Session 2
[6] Interactive patterns that lead to children's discursive changes in lessons comparing fractions. ***Keiko Hino*** *and Yuka Funahashi* (Japan). (IT)
[7] Assessing mental abstraction activities using eye-tracking techniques. ***Eivind Kaspersen*** *and Trygve Solstad* (Norway). (LO)
[8] Mathematics itself: reflections about an often neglected, but pivotal dimension. ***Michael Neubrand*** *and Carl von Ossietzky* (Germany). (LO)
[9] On the epistemological significance of contextualizing in mathematical cognition. ***Marcia M. F. Pinto*** *and Thorsten Scheiner* (Australia).
[10] Learning strategies used by high achieving and low achieving students in mathematics. ***Bishnu Khanal*** (Nepal).
Session 3
[11] The amazingly frequent, efficient, and flexible use of the subtraction-by-addition strategy in elementary school children's mental multi-digit arithmetic: A challenge for cognitive psychology and mathematics education. ***Lieven Verschaffel****, Joke Torbeyns, Gwen Verguts, and Bert De Smedt* (Belgium), (IT)
[12] Numerical processing profiles in children with varying degrees of arithmetical achievement. ***Nancy Estévez*** (Cuba), *Danilka Castro* (Chile), *Eduardo Martínez* (Cuba), *and Vivian Reigosa* (Uruguay). (LO)
[13] How proper use of mathematics can help students to build quantum physics thinking to learn the subject: simple harmonic oscillator. ***Jose Vieira Do Nascimento Junior*** (Brazil).
[14] A cognitive model of learning applied to data analysis of mathematics learning. ***Jairo Alfredo Navarrete*** (Chile).
[15] Exploring basic numerical capacities in children with varying degrees of arithmetical achievement. *Danilka Castro Cañizares,* ***Pablo Dartnell*** (Chile)*, and Nancy Estévez Pérez* (Cuba).

2.4. TSG-20 Program

Challenges associated with program organization arose from the online nature of the program (caused by a) the global pandemic, and time constraints associated with panel decision that all participants should be involved in all sessions. Safety nets were constructed to cater for technological problems that might arise. They included setting up multiple ways that a presentation could be uploaded, and construction of a TSG-20 Website where presentations could be made available, session information provided, and additional questions and discussions uploaded. Time constraints were reduced through email and website introductions to sessions and to invited researchers. The sessions and researchers are displayed below with capitalization of paper presenters. Discussion times were factored into each presentation and a short discussion time

occurred at the end of each session. Twenty to thirty participants attended each of the three sessions.

2.4.1. Session 1, July 13th. Host: Wenjuan Li

Processes of teacher learning, that can enhance student learning of mathematics, have been researched with various theoretical frameworks employed (including but not limited to cognitive, social, belief-based, and dispositional frameworks, and interconnections between various of these frameworks). Areas of research into influences on teacher change processes include but are not limited to professional learning models employed, types of pedagogical approaches under focus: teacher controlled or teacher-guided learning, and approaches enabling various degrees of student autonomy in the learning of mathematics. Presenters shared their recent work on new models or approaches to support teacher learning, and their theory or framework to analyze teacher change.

The invited talk[1] in Session 1 was given by Alison Superfine Castro, who is a Professor of Mathematics Education and Learning Sciences at the University of Illinois at Chicago. Her research interests focus primarily on studying and supporting mathematics teacher learning. Alison has developed different analytic approaches to study mathematics teacher educators and the knowledge needed to teach teachers. She has received various grants to design and study learning environments for mathematics teacher preparation courses and published extensively in the areas of mathematical knowledge for teaching, professional noticing, mathematics teacher's learning trajectory-based formative assessment practices. Alison is an active member of the international mathematics community. She is currently serving as an associate editor for the Journal of Mathematics Teacher Education and the EURASIA Journal of Mathematics, Science & Technology. In this ICME-14 TSG-20 invited talk, she presented new models and the design principles to support teacher professional learning.

The author gave the abstract as follows:

> New models for supporting teacher professional learning generate new conceptualizations of teacher learning, afford new designs for studying teacher learning over time, and situate teacher learning in problems of practice relevant to their own circumstances. In this paper, I describe two examples in which we engaged teachers in new models to support their professional learning, including examples of the various forms of inquiry we developed, as well as ways in which teachers engaged in the activities as part of these efforts. I then discuss a set of design principles underlying both examples. Finally, I discuss tensions that emerged from these efforts.

2.4.2. Session 2. July 16th, Host: Gaye Williams

This session focused around the learner — student learning processes, influences upon these learning processes, learning strategies employed, the degree of student autonomy in the learning process, the nature of the mathematical objects developed, and theoretical frameworks employed.

The invited talk[6] in Session 2 was developed by Keiko Hino with her colleague Yuka Funahashi. Hino is a Professor of Mathematics Education at Utsunomiya University in Japan. Her scholarly interests include the development of students' mathematical thinking through classroom teaching, international comparative study of the teaching and learning of mathematics, and mathematics teachers' professional development. Hino has produced many scholarly publications and undertaken positions to improve mathematics education, including as an editor of Japanese Primary and Lower Secondary School Mathematics Textbooks, as an external expert for Lesson Study in Mathematics, and as a member of the Editorial Board of MTED (Mathematics Teacher Education and Development Journal). Keiko Hino and Yuka Funahashi (Nara University of Education, Japan) have collaborated on research that informs the professional learning of teachers for more than ten years now, undertaking detailed analyses of problem-solving activity during mathematics lessons.

The abstract of the talk is as follows:

> The analysis presented in this paper examined the changes in the way children explained equivalent fractions and explored the teacher's key interactions that enabled such changes. Data from nine consecutive fifth-grade lessons in Japan taught by an experienced teacher were examined using a guided focusing pattern framework, from which it was found that the changes in the explanations were mostly in the focusing phase. The teacher's key interactive actions were classified into three categories: proposing focus, modifying focus, and narrowing focus. In particular, it was found that the teacher consistently attempted to change the children's focus from procedure to quantity and quantitative relationships using intervening language and by evoking discursive rules.

2.4.3. Session 3. July 17th, HOST: Pablo Dartnell

This session is built mostly around TSG-20 ICME-14's 3rd subtheme: The Science of Learning, although it necessarily has some components from the other two subthemes. The science of learning is dedicated to furthering the scientific understanding of learning processes for the purpose designing and implementing learning innovations to increase learning opportunities. Among the presentations scheduled for this session, our invited speaker shares findings about surprisingly frequent and efficient use among Belgian elementary students of a mental subtraction strategy. In addition, results are shared about basic cognitive numerical capacities and their relationship with arithmetic difficulties, conducted in two different Latin American countries; relationships between the use of mathematics and the building of knowledge in quantum physics; and a proposed cognitive model of learning with implications for the analysis of data regarding the learning of mathematics.

The invited talk[11] in this session was given by Lieven Verschaffel, who is a full professor in Educational Sciences, and director of the Center for Instructional Psychology and Technology (CIP & T) at the Katholieke Universiteit Leuven, Belgium. His research work in Mathematics Education covers a wide variety of topics, many of

them related to TSG-20, such as problem solving, strategy choice and change, conceptual change, metacognitive and affective aspects of learning, and early and elementary mathematical education. The quality of his research work has led to him. receiving many awards, invitations as plenary lecturer, and member of plenary panels of many international conferences, including some previous versions of ICME. In addition, he. has become a member of many editorial boards. On this occasion he presented the result of two studies (conducted in collaboration with other researchers from KU Leuven) dealing with a mental subtraction strategy used by elementary school students.

The abstract of the talk is as follows:

> In two related studies — a first study with 6th grade elementary school children and a second study with children from 4th until 6th grade of elementary school, we investigated the use of the subtraction-by-addition strategy in children with different levels of mathematics achievement. In doing so, we relied on Siegler's cognitive psychological model of strategy change, which defines strategy competencies in terms of four parameters — strategy repertoire, distribution, efficiency, and selection — and the choice/no-choice method, which is essentially characterized by offering items in two types of conditions — choice and no-choice conditions. In both studies, children of different mathematics achievement levels solved multi-digit subtraction problems in the number domain up to 1,000 in one choice condition (wherein they could choose between direct subtraction or subtraction by addition on each item) and two no-choice conditions (wherein they had to use either direct subtraction or subtraction by addition on all items). Distinction was made between two types of subtraction problems: problems with a small versus large difference between minuend and subtrahend. Although mathematics instruction only focused on applying direct subtraction, most children reported using subtraction-by-addition in the choice condition. Subtraction-by-addition was also applied surprisingly frequently and efficiently, particularly on small-difference problems, and children flexibly fitted their strategy choices to both numerical item characteristics and individual strategy speed characteristics. Interestingly, these results were obtained for children of all grades and all mathematical achievement levels. These remarkable findings — both from a cognitive psychological and a mathematics educational perspective — add to our theoretical understanding of children's strategy acquisition and challenge current mathematics instruction practices that pay exclusive attention to direct subtraction.

3. Reflections and Future Directions

Time for discussion of future directions was limited by participants' interest in continuing discussion of research from the third session. Post-session reflections from participants (see italics below) indicated intended future directions though. Slight changes to quotes were made to increase clarity. Names of invited speakers or presenters are used to refer to studies in this section.

In Session 1, Alison Superfine Castro[1] stimulated new thinking about models to support teacher professional learning: *I enjoyed the presentation of Dr Castro very much, it was an eye-opener for future research paths*. Participants connected other research from this session to her work: *Alison Castro Superfine's contribution was quite informative to me, as was Williams'*[5] *and other presentations (e.g., Berisha's*[2]*). Collaboration seems to be a key point in teacher education.*

In Session 2, Keiko Hino's presentation[6] was appreciated by participants for the detailed way it examined interactions between the teacher and the students, and diagrammatic representations communicating this: *The way the research and the teaching in the classroom were analysed was very good*, and *the diagram helped in seeing the large amount of time the classroom teaching dedicated to interactions between teachers and students*. This research was referred to in subsequent presentations and reflections. Comparisons were made between Pinto's *proposal*[9] *for working with fractions, developed from the perspective of contextualizing* and Keiko Hino's presentation *typical of Japanese teaching undertaken without the usual context (real world etc) but still in the reflection mode*. Pinto's team intend to use Keiko Hino's work to extend their *thinking about other possible approaches to equivalence of fractions.*

The interconnected nature of most presentations in Session 2 was also recognised: *the research presented is interwoven in many senses. Four presentations focused on strategies of learning or modes of learning (or a specific mode of learning), to inform teaching or educational policies. Neubrand*[8] *and Kaspersen*[7] *focused on abstraction but differed in their conceptions of the interpretative model built. Further discussion of these two approaches and methodologies should be productive. Pinto*[9] *and Neubrand*[8] *proposed different categories for organizing understanding of the same phenomena — the leaning of maths. Both also attempted to avoid dichotomies*.

In Session 3, Lieven Verschaffel's presentation[11], participants were *surprised by the prevalence of use of the Subtraction-by-Addition method and the accuracy and speed of responses found in Belgium*. This presentation raised questions for future research including a*) could these results be replicated in other places* and b) *might there be alternative strategies that could be employed for other mathematical procedures, that could produce similarly strong results?* Questions for future research were also raised by Dartnell's presentation[15]: *Given that very low achievement in mathematics can have a variety of causes—not always related to a disability—what might be found if students with Mathematics Learning Disabilities were the focus of such research?*

In Summary, sessions for TSG-20 were vibrant and extended the thinking of various participants in different ways. Our thanks to all participants.